been dredged up, showing that it was synchronous or closely continuous with the so-called preceding Age of Stone.

Dr. Coates observed that, as he apprehended, well-known historical documents were omitted to be noticed, which yet bear upon these subjects. The transition from the brass to the iron age is indicated by Horace, and appears to have taken place, as in the migrations of the Dorians, with few if any battles. The heroes of the Iliad fight with large stones. A high state of civilization existed at the same time, as in the mention of Phoenicea, Mycenæ, and Tyre. A population being driven into a lake, and founding even a mighty city there, is exemplified by Mexico, according to the hieroglyphic records abstracted by Clavigero. Venice, also, though not situated in a lake, is a case of much analogy.

Dr. Coates reminded the Society of the general omission, as a fact in the history of the later geological age, of the narrative in Genesis of the destruction and deep depression of the cities and cultivated plain of Sodom, although preceded by the account of a great battle, with mention made of nine names of cities and eight of commanders, and connected with the account of Abraham, Lot, and Melchisedek.

Pending nomination No. 525, and new nominations Nos. 526, 527, 528, were read.

And the Society was adjourned.

Stated Meeting, September 16, 1864.

Present, eight members.

Mr. Lea, Vice-President, in the Chair.

Letters accepting membership were received from Mr. J. F. Kirk, dated Dorchester, Massachusetts, August 25th; and from Dr. Louis Stromeyer, dated Hanover, July 25th, 1864.

A letter resigning membership was received from Mr. E. E. Law, dated Philadelphia, September 10th, 1864. On motion his resignation was accepted.

A letter asking to be excused from writing another obituary notice of Mr. Quincy was received from Mr. E. Everett, dated Boston, August 27th, 1864. On motion Mr. Everett was excused.

Photographs for the Album were received from Dr. W. S. W. Ruschenberger, Professor W. Chauvenet, Dr. L. Stromeyer, and Dr. Isaac Hays.

Letters of acknowledgment were received from the Asiatic Society of Bengal, Calcutta, October 3d, 1863; the Corporation of Harvard College, August 22d, and Captain Gilliss, Washington, August 20th, 1864.

Letters of envoi were received from the Smithsonian Institution, and Mr. J. W. Irwin, of New York City.

Donations for the Library were received from Dr. Stromeyer; the Annales des Mines; the London Society of Antiquaries; Harvard College; Silliman's Journal; the Brooklyn Mercantile Library Association; Messrs. Blanchard & Lea, and Mr. Eli K. Price, of Philadelphia.

Mr. Lea made a communication of a discussion of "Prime Right-Angled Triangles and $\sqrt{2}$," from a private letter addressed to him from Dr. James Lewis, of Mohawk, New York.

Prime Right-Angled Triangles, and $\sqrt{2}$.

In any R. A. Triangle, let $H =$ hypothenuse, $P =$ perpendicular, $B =$ base.

Then $H^2=P^2+B^2$; whence $H^2-P^2=B^2$. H^2-P^2 is the product of two factors, $H+P$ ($=a$) and $H-P$ ($=b$). Accordingly,

$$H^2-P^2=(H+P)\times(H-P)=ab=B^2, \quad \text{and } \sqrt{ab}=B.$$

$$a+b=(H+P)+(H-P) \quad =2\ H \quad \text{and } \tfrac{a+b}{2}=H.$$

$$a-b=(H+P)-(H-P) \quad =2\ P \quad \text{and } \tfrac{a-b}{2}=P.$$

The radical sign before ab implies that the terms a and b are squares; the fractional expressions $\frac{a+b}{2}$ and $\frac{a-b}{2}$ implies that those terms should be multiplied by 2. Substituting for the terms a and b, others that meet these indications, viz., $a=2N^2$ and $b=2S^2$, the sides of R. A. Triangles have the following general expression: $H=N^2+S^2$, $P=N^2-S^2$, $B=2NS$. If the terms N and S be any whole numbers, their expansion as indicated in the formula, will evolve the sides of Prime Right-Angled Triangles (prime, in the sense that the triangles thus evolved may be analyzed without the introduction or suppression of factors). If in the above formula for prime R. A. Triangles, x+y be substituted for N, and y for S, the following formula will appear: $H=x^2+2\times y+2y^2$, $P=x^2+2\times y$, $B=2\times y+2y^2$. This second formula will be chiefly referred to in the following summary:

DECOMPOSED FRACTION OF THE SQUARE ROOT OF 2.

$$\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+1}}}} \quad \&c.=\underset{(\triangle}{\tfrac{1}{1}}=\tfrac{3}{2}=\underset{\triangle}{\tfrac{7}{5}}=\tfrac{17}{12}=\underset{\triangle}{\tfrac{41}{29}}=\tfrac{99}{70}=\underset{\triangle)}{\tfrac{239}{169}} \;\&c. \quad \triangle=\frac{x}{y}=\frac{B+P}{H}.$$

$$\frac{B+P}{H}=\frac{x}{y}=\sqrt{2}\text{ approx.}$$

x		y
1	△	1
3		2
7	△	5
17		12
41	△	29
99		70
239	△	169

The decomposed fraction of $\sqrt{2}$, when resolved into a series of numerators and denominators of common fractions $(\frac{x}{y})$ will present the values in the annexed table, column x embracing numerators, and column y the corresponding denominators. The first two terms, or INITIALS of the series, being found, succeeding terms may be found by additions, observing the following relations, $x+y=y'$, $y+y'=x'$, $x'+y'=y''$, $y'+y''=x''$, &c., continuously; or, $y+2y'=y''$, $y'+2y''=y'''$, $y''+2y'''=y''''$, &c.

In this series it will be seen that each alternate fraction (△) embraces a triangle in the form $\frac{B+P}{H}$ in each one of which triangles is a *common characteristic*, having the expression $B-P=\pm1$.

An analysis of the several triangles of this series, by means of the formula embracing the terms x and y, will reproduce the series of values of x and y respectively, as given in the table. This is the only series which will reproduce its radical elements, for the reason that $\sqrt{1}=1$; and the root of no other quantity than 1 is equal to itself. If the several triangles be analyzed by the formula embracing the terms N and S, N and S will successively reproduce the series of denominators y, N being in advance of S.

The general character of this and any similar series of triangles, suggests the expression "$\frac{B+P}{H}=\sqrt{2}$ approximately." Other series of triangles similarly derived from different *initials* will confirm this suggestion.

The intermediate alternate terms in the series (not distinguished by the sign $\triangle$) are only approximately R. A. triangles, having the character which may be inferred from the following expression. $B^2+P^2=H^2\pm 1$.

The preceding table, derived from the *initials* "1, 1," presents a series of approximate common fractions of the square root of 2, a quantity that can only be approximately expressed in limited terms. As expressions of the value of the root of 2, the successive fractions present discrepancies which have peculiar relations to $\sqrt{1}$ or 1. Other series similarly derived from other initials, will, in a similar manner, be approximate fractions of $\sqrt{2}$, but each series will have its characteristic discrepancy, which is related to the series as $\sqrt{1}$ or 1 is to the series just considered. This discrepancy will appear in any series as the quantity D in the expression $B-P=\pm D$, which expression is the *characteristic* of all the R. A. *triangles* of a series.

$\frac{x}{y}=\sqrt{2}$ approximately

x		y
1	$\triangle$	2
5		3
11	$\triangle$	8
27		19
65	$\triangle$	46
157		111
379	$\triangle$	268

The annexed series exhibits proportions similar to those of the preceding. The characteristics of the triangles in the alternate terms ($\triangle$) in the form $\frac{B+P}{H}$, is $B-P=\pm\sqrt{7}$. The *approximate* R. A. triangles (not designated by the sign $\triangle$) may be characterized by the expression $B^2+P^2=H^2\pm\sqrt{7}$. The successive fractions $\frac{B+P}{H}$ regarded as approximations to $\sqrt{2}$, present discrepancies which are related to $\sqrt{7}$ or 7 as the discrepancies of the preceding table are related to $\sqrt{1}$ or 1.

The triangles evolved by expanding the successive values of x and y, as in the formula embracing those terms, have the characteristics $B-P=\pm 7$.

This table having been derived from the initials "1, 2," by additions, may be continued backwards by subtractions, which will develop a series of terms, among which negative quantities will appear. The first pair of terms (x and y) in which a negative quantity appears, may be regarded as the *initials* of a new series, *correlative* to that from which it is derived. The *initials* being found may both be regarded as positive, and the series extended by additions, as in the preceding instances.

$x-y=y_{,}$. $\quad y-y_{,}=x_{,,}$, $x_{,}-y'=y_{,,}$, &c. $\qquad x_{,}=+3$. $\quad y'=-1$, the initials of the series.

$\frac{x}{y} = \sqrt{2}$ approximately

x		y
3	—	1
5	△	4
13	—	9
31	△	22
75	—	53
181	△	128

The annexed series is the *correlative* of the last preceding, and is in all respects similar, except that it is derived from different initials. Either series being given, the other may be found in the manner suggested.

In any case when a series has a *correlative*, it may be found in the same manner. In any instance in which D in the expression $B - P = \pm D$, is a prime number, or a multiple of a prime number, and the triangle to which the expression refers, is a Prime Right-Angled Triangle, as previously defined, an analysis of the triangle by the formula embracing the terms x and y, will give values for x and y, which may be extended into a series, which series has its correlative, as in the preceding instance. If D is found to embrace several factors which are prime numbers, each one of those prime factors may be found to give rise to two series of values for x and y, which will be correlative to each other, so that there will be twice as many series of values of x and y as there are prime factors in D, and accordingly twice as many series of R. A. Triangles in which $B - P = \pm D$ as there are prime factors in D.

If D is the square of a prime number, there will be three series of values for x and y, two of which will be *correlative* to each other, the initials of the third being $x = y = \sqrt{D}$.

Other generalizations might be suggested co-ordinate with these, which, however, are yet incomplete, and are reserved for further consideration.

If any prime R. A. Triangle be resolved into the fractional form $\frac{B+P}{H}\left(=\frac{x}{y}\right)$ and a series of fractions be derived therefrom by additions or subtractions, as in the preceding illustrations, the alternate corresponding values of x and y in the series will embrace a Prime R. A. Triangle in the form $\frac{B+P}{H}\left(=\frac{x}{y}\right)$, and all such triangles in the series will have the same value for D in the expression $B - P = \pm D$. If the series of values of x and y thus tabulated be expanded into a series of triangles (by means of the formula embracing x and y) the triangles thus evolved will be characterized by the expression $B - P = \pm D^2$, (D referring to its value in the former instance.)

Any whole numbers whatever, when used as the initials of a series under x and y, as in the preceding illustrations, will develop a series of numerators x and denominators y, of *common fractions ap-*

proximating $\sqrt{2}$, each alternate numerator embracing $B+P$ and its corresponding denominator H of some R. A. Triangle, in which the difference of B and P will be the square root of some whole number which will characterize the series; and the numerical value of $B—P$, will have the same relation to the series that $\sqrt{1}$ has to the series derived from the initials "1, 1" in the first illustration. From the various considerations here presented, is derived the general inference $\frac{B+P}{H}=\sqrt{2}$ approximately.

JAMES LEWIS,
Mohawk, N. Y., August 31, 1864.

Mr. Chase read a communication "On the Comparative Fitness of Languages for Musical Expression;" presenting the results of an investigation that had been suggested by the remarks of Mr. Foulke, at a previous meeting of the Society.

The fitness of any language for musical expression, depends, not on the number and character of the letters, but on the sounds that are expressed by their several combinations. I have, therefore, thought that it would be interesting to analyze the sounds, and to institute a variety of comparisons between Italian, French, English, and German, in order to determine as nearly as possible the precise nature of their harmonic differences.

In making my comparisons, I selected a number of the principal poets in each language, and examined a few passages from each. From the aggregates I framed the following Tables:

1. Number of sounds of each description in 10,000 syllables.

Sounds.	Italian.	French.	German.	English.
Vowel, . .	10,207	10,355	10,778	11,439
Nasal, . .	1,957	2,928	3,547	2,952
Liquid, . .	2,806	2,883	3,419	3,348
Sibilant, . .	1,597	2,436	2,113	2,643
Labial, . .	1,389	2,385	1,899	1,982
Dental, . .	2,532	2,132	3,626	4,266
Guttural, . .	969	863	2,341	1,351
	21,457	23,982	27,723	27,981

2. Proportion of sounds in a given number of ideas.

Sounds.	Italian.	French.	German.	English.
Vowel, . .	10,207	8,845	9,765	9,579
Nasal, . .	1,957	2,502	3,214	2,472
Liquid, . .	2,806	2,467	3,098	2,804
Sibilant, . .	1,597	2,084	1,914	2,213
Labial, . .	1,389	2,041	1,720	1,660
Dental, . .	2,532	1,825	3,285	3,572
Guttural, . .	969	738	2,121	1,131
	21,457	20,502	25,117	23,431

3. Number of sounds of each description in 10,000 sounds.

Sounds.	Italian.	French.	German.	English.
Vowel, . .	4,757	4,312	3,888	4,088
Nasal, . .	912	1,221	1,279	1,055
Liquid, . .	1,308	1,204	1,233	1,197
Sibilant, . .	744	1,017	762	945
Labial, . .	648	996	685	708
Dental, . .	1,180	890	1,308	1,524
Guttural, . .	451	360	845	483
	10,000	10,000	10,000	10,000

It appears, therefore, that

1. In a given number of syllables, English has the greatest number of sounds, and Italian the least.

2. In the expression of a given number of ideas, German uses the greatest number of sounds, and French the least.

3. Italian is the richest in the most musical sounds, or the vowels and liquids,—German in the nasals and gutturals,—French in the sibilants and labials,—and English in the dentals.

4. In regard to the harshest and least musical sounds, German has an excess of gutturals, French of sibilants, and German of gutturals and sibilants combined.

5. German has the greatest proportion of mute, and the smallest proportion of vocal and semi-vocal sounds, and is, therefore, the least musical of the three languages.

Mr. Chase also read a communication "On Certain Primitive Names of the Supreme Being."

The resemblance between Algonquin "Manitou," Chinese mang taou, Egyptian ma ntr, Latin magnus deus, Greek μέγας θεός, and Sanscrit maha deva, to which I made casual reference in a former communication (Trans. A. P. S., New Series, vol. xiii, p. 61), has seemed to some of my friends sufficiently interesting and important to justify a more minute and analytical exposition. The last three forms are confessedly cognate; the resemblance between the others is at least equally striking, and were it not for the wide geographical separation of the nations, and the absence of any direct evidence of intercourse, we should naturally suppose that they were derived from the same original source. But since it would be difficult to select three languages that are less likely to have sprung from a common parentage, it is reasonable to expect that the coincidence will be severely criticized by those whose prejudices forbid a belief in the unitary origin of man, and that little regard will be paid to the mathematical probability of any hypothesis that may be supposed to weaken its significance.

The Chinese corresponds precisely in meaning with the Aryan forms, but the extent of the correspondence in the Algonquin and Egyptian words, is somewhat uncertain. The probability that it is equally complete in Algonquin, is strengthened by the considerations that, 1, the Indian tribes generally speak of the Deity as the "Great Spirit;" 2, the Shyenne word for spirit is mahio; 3, the Algonquin forms mechekelo, miss, michau,—Blackfoot omuku, omaesin,—Cushna muck,—Shyenne tsimahaa,—all signify "great," and all contain the root ma, variously modified, as in C. mang, S. ma̧h, L. magn-, mag-, Greek μεγ-, μακ-, E, mass, much, many, &c., &c. In Egyptian, ma ntr is properly "the true God," naa ntr being equivalent to "magnus deus." But m and n are often interchanged; the root na or naa is found in Arapoho benasa, *large*, naathia, *so large;* some of the subordinate meanings of ma are retained in Egyptian mh or mah = Ch. mang, *to fill*, and in mak = L. mag-, *to rule;* and a probable association between the ideas of truth and greatness is shown by S. uru, G. ur, L. verus, E. very, &c.

It is probable that tr and taou are both compounds, and that they may have both been originally identical in meaning, and perhaps also in form, is shown by S. tr, trai, *to conquer, to preserve, to guard;* Pawnee terahu or tidihu, *great*, terawa, *god* (Cfr. also Eg. ra, *the sun;* S. ravis, *the sun*, râdȷ, *to govern*, racſ, *to guard*, I'svaras, *God;* Crow, isa, *large*).

The Chinese and Indian languages appear to furnish a clue to some of the other primitive names of the Supreme Being. The root aou, *to shine* (which is in its organic formation, as well as in its significance, almost identical with r), is found nearly pure in C. haou, *appearance of the sun rising, the light of the heavens;* M. 3252–4, gaou, aou or yaou, *the light of the sun*, M. 2932; Iowa, hawe, *day;* Eg. auu, *splendor*, aui, *to glorify;* H. אור, *light;* L. aurora, Y. auro, *morning*. The same root is traceable, with some slight modifications, in C. ou or woo, *bright*, M. 11747, yaou, *very white, the splendor of the sun*, M. 11965, 11973, yuh, *the splendor of the sun*, M. 11870, heu or yu, *the air extending itself, great, warmth, sunlight, morning*, M. 3738–41, 3756–7, tsaou, *morning*, M. 10540, chaou, *the splendor of the sun, daylight*, M. 349, 1459, keaou, *white*, M. 5578, we or wei, *the light of the sun*, M. 11661, wang, *the sun going forth and shedding his illuminating beams*, M. 11620, paou, *the fierce rays of the sun;* D. wi, *the sun*, aŋpao, *dawn, daylight;* Assiniboin, aumpa, *day;* Pawnee, tewauwaupits, *lightning;* Shyenne, iwonit, *the moon rising*, iniwooniyots, *day breaking*, wowoiwo, *morning star;* Algonquin, kayshoh ("the mighty Yoh" = magnus Jov- = Shyenne mah-io), *sun, day;* S. dʝval, *to blaze*, dyu, *to shine*, dyaus, *the bright heaven;* Gr. ἔως, L. Eurus, C. heuh, *the light of the morning*, M. 3857.

Material existence is principally manifested through the medium of light, and accordingly we find C. we or wei, *to be*, M. 11640, yew, *existence*, M. 12107; Eg. au, *to be;* Alg. iah, iau, *to be, to do, to have;* Chaldee, yáo;* H. יה, יהוה; Ger. wesen; E. was. Prof. Max Müller has well shown the connection of deus, deva, Θεός, Ζεύς, with S. dyaus, and the resemblance of the Hebrew ה, יהוה on the one side to Alg. iah, iau, and on the other to L. Ju-, Jov-, has been pointed out by different writers, but I think no one has shown how readily all these forms may be connected through the Chinese heaou, *vapor, breath*, M. 3556, 3580; yaou, *the glory of the sun*, M. 11965; yew, *existence;* teaou, *the sovereign or watchful yaou*, M. 9992, 10004; teaou, *to look to a distance, a species of dragon*, M. 10031, 10045;† taou, *the principle from which heaven, earth, man, and all nature emanates*. According to Morrison, "Taou, in the books of Laou tsze, is very like the *Eternal Reason*,

* See Bunsen's Egypt, vol. iv, p. 193–4.

† Cfr. S. drç, Gr. δέρκω, L. draco (= "the Watcher"). The dragon is the badge of the Emperor of China, and is embroidered or painted on his standards, "in the manner of the ancient Scythians, Parthians, Persians, and Romans." The basilisk was also the emblem of the sun-god and of the monarch in Egypt.

of which some Europeans speak; *Ratio* of the Latins, and the *Logos* of the Greeks." The resemblance between Mexican Teo and Greek θεός, has been often noticed, but it has usually been dismissed as a merely accidental, though curious coincidence. In comparison with the analogues here given, it assumes a new importance.

The association of the ideas of whiteness, purity (πῦρ), brilliancy, divine glory, and sacrifice, may perhaps account for such resemblances as C. yang, *a sheep or goat, fire burning fiercely, bright, splendid, the sun, male, the superior of the two material principles into which, according to the Chinese, chaos was divided;* S. yas, *light, lustre,* adjas, *a goat,* yadj, *to sacrifice,* agni, a̧gati, *fire;* L. agnus; Gr. αἴξ; Y. ake, *a goat,* agutaŋ, *a sheep,* ako (= Eg. ka), *male,* eraŋ, *a sheep* (raŋ, yaŋ, saŋ, *to shine, to burn, brilliantly*); D. saŋ, *whitish or yellowish,* agu, *to burn on or on account of anything,* wiyakpa, iyoyaŋpa, *to shine,* aŋpao, *day,* takiŋ-wanuŋ-yaŋ-pi (= "deer-accidentally-domesticated*-flock") *sheep;* Mandan, aŋsakte (= "the great aŋsa"), *mountain sheep.* The Shyennes call the sheep "the white deer."

Some of the Chinese religious expressions appear to furnish traces of the remains of an early inspiration, as well as a parallelism of thought that is indicative of a common origin. For example, by combining the two characters which represent *my* and *sheep*, the Chinese form the character for e, *good, right, suitable, righteousness.* Morrison says (under the word "light"), "The Budhists speak of a light within; thus of the principles of the Kin Kang King, they say, 'This sacred book is originally possessed by all mankind in their own nature, unperceived by themselves. When they are awakened to know their own hearts, they are assured of the internal scripture. Having the light within, they do not, like the men of the world, seek for Budh outside their own persons, nor seek for a scripture externally, but rouse the internal mind, and adhere to the internal mental scripture.' Does not this language resemble that of the Friends, called Quakers?"

Some of the latest triumphs of physical science have led to the revival of beliefs nearly identical with the intuitive or inspired perceptions of our early ancestors, as manifested in their worship of the mysterious Agency that controls the Universe. Thus we find in

* The Chinese speak of the lew chŭh, the "six domestic animals," ma, *the horse* (Cfr. G. mahre, E. mare), new, *the cow* (Cfr. Sw. nöt, Dn. nöd, W. cnud, E. neat), yang, *the sheep,* ke, *the fowl* (the "caller"), keuen, *the dog* (Cfr. S. çvan, Gr. κύων), che, *the hog* (Cfr. F. cochon).

Tyndall's lectures on "Heat considered as a mode of motion" (First American Edition, p. 446, sqq.), such expressions as the following: "Every mechanical action on the earth's surface, every manifestation of power, organic and inorganic, vital and physical, is produced by the sun. . . . . He blows the trumpet, he urges the projectile, he bursts the bomb. And remember, this is not poetry, but rigid mechanical truth. He rears, as I have said, the whole vegetable world, and through it the animal; the lilies of the field are his workmanship, the verdure of the meadows, and the cattle upon a thousand hills. He forms the muscle, he urges the blood, he builds the brain. His fleetness is in the lion's foot; he springs in the panther, he soars in the eagle, he slides in the snake. He builds the forest, and hews it down; the power which raised the tree and which wields the axe being one and the same. . . . The sun digs the ore from our mines, he rolls the iron, he rivets the plates, he boils the water, he draws the train. He not only grows the cotton, but he spins the fibre and weaves the web. There is not a hammer raised, a wheel turned, or a shuttle thrown, that is not raised, and turned, and thrown by the sun." No Chinese Bonze, no Hindoo Brahmin, no Persian Fire-worshipper, no Egyptian, Grecian, or Roman priest, no Indian medicine-man, could have discoursed in more eloquent language of the power of the "Mighty Ra" or "Yau," and none perhaps, with less danger of inculcating the belief, that the mere inert material nature can exert that all-controlling power which is essentially spiritual, and can spring only from a Supreme Intelligence.

Pending nominations Nos. 525, 526, 527, 528, were read.

And the Society was adjourned.

Stated Meeting, October 7, 1864.

Present, sixteen members.

Dr. Wood, President, in the Chair.

A letter accepting membership was received from Mr. Joseph Harrison, dated Philadelphia, September 24th, 1864.

A letter acknowledging the receipt of publications was re-